AQA

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MFP2

Unit Further Pure 2

Monday 13 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

condensed

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1 (a) Draw on the same Argand diagram:

(i) the locus of points for which

$$|z - 2 - 5\mathrm{i}| = 5$$ *(3 marks)*

(ii) the locus of points for which

$$\arg(z + 2\mathrm{i}) = \frac{\pi}{4}$$ *(3 marks)*

(b) Indicate on your diagram the set of points satisfying both

$$|z - 2 - 5\mathrm{i}| \leqslant 5$$

and

$$\arg(z + 2\mathrm{i}) = \frac{\pi}{4}$$ *(2 marks)*

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2 (a) Use the definitions of $\cosh\theta$ and $\sinh\theta$ in terms of e^{θ} to show that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(x - y)$$ *(4 marks)*

(b) It is given that x satisfies the equation

$$\cosh(x - \ln 2) = \sinh x$$

(i) Show that $\tanh x = \frac{5}{7}$. *(4 marks)*

(ii) Express x in the form $\frac{1}{2}\ln a$. *(2 marks)*

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3 (a) Show that

$$(r + 1)! - (r - 1)! = (r^2 + r - 1)(r - 1)!$$ *(2 marks)*

(b) Hence show that

$$\sum_{r=1}^{n}(r^2 + r - 1)(r - 1)! = (n + 2)n! - 2$$ *(4 marks)*

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4 The cubic equation

$$z^3 - 2z^2 + k = 0 \qquad (k \neq 0)$$

has roots α, β and γ.

(a) (i) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. *(2 marks)*

(ii) Show that $\alpha^2 + \beta^2 + \gamma^2 = 4$. *(2 marks)*

(iii) Explain why $\alpha^3 - 2\alpha^2 + k = 0$. *(1 mark)*

(iv) Show that $\alpha^3 + \beta^3 + \gamma^3 = 8 - 3k$. *(2 marks)*

(b) Given that $\alpha^4 + \beta^4 + \gamma^4 = 0$:

(i) show that $k = 2$; *(4 marks)*

(ii) find the value of $\alpha^5 + \beta^5 + \gamma^5$. *(3 marks)*

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5 (a) The arc of the curve $y^2 = x^2 + 8$ between the points where $x = 0$ and $x = 6$ is rotated through 2π radians about the x-axis. Show that the area S of the curved surface formed is given by

$$S = 2\sqrt{2}\pi \int_0^6 \sqrt{x^2 + 4}\,\mathrm{d}x$$ *(5 marks)*

(b) By means of the substitution $x = 2\sinh\theta$, show that

$$S = \pi(24\sqrt{5} + 4\sqrt{2}\sinh^{-1} 3)$$ *(8 marks)*

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6 (a) Show that

$$(k+1)(4(k+1)^2 - 1) = 4k^3 + 12k^2 + 11k + 3$$ *(2 marks)*

(b) Prove by induction that, for all integers $n \geqslant 1$,

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{1}{3}n(4n^2 - 1)$$ *(6 marks)*

Turn over ▶

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7 (a) (i) Use de Moivre's Theorem to show that

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

and find a similar expression for $\sin 5\theta$. *(5 marks)*

(ii) Deduce that

$$\tan 5\theta = \frac{\tan\theta(5 - 10\tan^2\theta + \tan^4\theta)}{1 - 10\tan^2\theta + 5\tan^4\theta}$$ *(3 marks)*

(b) Explain why $t = \tan\frac{\pi}{5}$ is a root of the equation

$$t^4 - 10t^2 + 5 = 0$$

and write down the three other roots of this equation in trigonometrical form. *(3 marks)*

(c) Deduce that

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5} = \sqrt{5}$$ *(5 marks)*

END OF QUESTIONS

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